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A STUDY ON COMPUTATIONAL ANALYSIS OF G-AUTO COMMUTING PROBABILITY IN FINITE GROUPS

Gunjan[1], and Jakhar, Manjeet Singh[2]

[1]Research Scholar, Department of Mathematics, NIILM University Kaithal Haryana

[2]Associate Professor, Department of Mathematics, NIILM University Kaithal Haryana

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Abstract

The chance that a pair of elements randomly selected from a group G commute under some automorphism of G is measured by the G-auto commuting probability in finite groups. This study presents a comprehensive computational analysis of G-auto commuting probability in finite groups. By leveraging algorithms and computational methods, we explore the structural insights and patterns revealed by the interaction between group elements and automorphisms. Our approach assesses groups of varying sizes and complexities, examining how the probability changes with different group properties and automorphism structures. The results provide a foundation for understanding the intrinsic symmetry and dynamics of finite groups, offering potential applications in group theory, coding theory, and other areas where group symmetry plays a key role.

Keywords: Automorphisms, Commutativity, Finite groups

Introduction

In the realm of group theory, the study of finite groups and their automorphisms is fundamental for understanding the underlying structures and symmetries within these groups. Automorphisms are mappings that preserve the group operation and play a crucial role in exploring group symmetry and behaviour. One way to measure the interaction between group elements and their automorphisms is through the G-auto commuting probability.

The likelihood that an automorphism would cause a pair of elements from a finite group G to commute is known as the G-auto commuting probability. This concept extends the idea of commutation, providing a more nuanced view of group dynamics under automorphisms. By studying this probability, researchers can gain insights into the inherent regularity and structure of groups.

This study aims to conduct a computational analysis of G-auto commuting probability in finite groups. We use algorithms and computational methods to calculate the probability across various groups, exploring how it changes with different group properties and automorphism structures. By examining groups of different types and sizes, we aim to identify patterns and relationships that provide a deeper understanding of group symmetry and its implications.

The study's findings may help other branches of mathematics where group symmetry plays a significant role, such representation theory and coding theory. Furthermore, by providing a fresh viewpoint on the relationship between groups and their automorphisms, this study may be used as a basis for further investigations into the characteristics and categorization of finite groups. We anticipate that our computer approach will provide important new understandings of the nature of finite groups and their intricate dynamics.

Theory of G-Auto commuting probability in Finite Groups

The theory of G-auto commuting probability in finite groups provides a framework to study how elements of a group commute under the influence of its automorphisms. This concept ties together the structure of a group with the nature of its automorphisms, offering insights into the underlying symmetry and interactions within the group.

Key Concepts

1. **Groups and Automorphisms**: The mathematical concept of a group is a set whose binary operation satisfies the group axioms, which include closure, associativity, the identity element, and inverse elements. A bijective homomorphism from the group to itself, or simply a mapping that maintains the group structure, is called an automorphism of the group. A group's automorphism group is the collection of all of its automorphisms.
2. **Commutativity and Auto commutativity**: In a group, two elements commute if their product is independent of the order in which they are multiplied (i.e., xy=yx). The concept of auto commutativity extends this idea to include the influence of automorphisms: two

elements gg and xx are G-auto commutative if there is an automorphism φ such that $gx=xg\varphi$.

3. **Auto commuting Probability**: A group's G-auto commuting probability estimates the chance that a pair of randomly selected group elements will be auto commutative under an automorphism. This probability provides a way to quantify the extent to which a group's elements interact symmetrically under its automorphisms.

Theoretical Implications

1. **Group Symmetry and Regularity:** A high G-auto commuting probability suggests a group with a high level of symmetry and regularity. Such groups often have predictable structures and may be easier to analyse.
2. **Structure of the Automorphism Group:** By studying the G-auto commuting probability, one can gain insights into the automorphism group of G and its action on the group. This can help identify important structural aspects of the group.
3. **Classification and Invariant Properties:** The G-auto commuting probability can be used as an invariant measure to classify different groups. Groups with similar probabilities may share certain structural characteristics.
4. **Applications:** Understanding G-auto commuting probability can have applications in other areas of mathematics, such as group actions, representation theory, and coding theory, where group symmetry plays a key role.
5. **Comparisons with Other Groups:** The G-auto commuting probability can be compared across different groups to identify similarities and differences in their structures and automorphism groups.

Challenges and Considerations

1. **Computation**: Calculating the G-auto commuting probability can be challenging, particularly for larger groups or groups with complex automorphism structures.
2. **Dependence on Automorphism Group**: The probability is directly influenced by the structure and properties of the automorphism group of G, which can vary widely across different groups.
3. **Relation to Other Group Properties**: The G-auto commuting probability may be related to other group properties, such as the commutativity degree, which measures the likelihood of two elements commuting.

In summary, the theory of G-auto commuting probability in finite groups offers a unique perspective on group interactions under automorphisms. It helps bridge the gap between group theory and its automorphism group, revealing the intricacies of group structure and symmetry. This measure can be a powerful tool for researchers studying finite groups and their automorphisms.

On The g-Auto commuting Probability of a Finite Group

An element g of a finite group G is defined. The g-auto commuting probability of G, Prg(G, Aut(G)), is introduced and studied in this chapter. We say that Prg(G, Aut(G)) is the chance that, given a randomly selected pair of elements from G and Aut(G), their autocommutator is equal to g. Presumably, when g equals 1, Prg(G, Aut(G)) equals Pr(G, Aut(G)). Keep in mind that for any set G = L(G) and g = 1, Prg (G, Aut(G)) = 1 and only if K(G) = {1}. Further, the inequality Prg (G, Aut(G)) = 0 holds only when g /∈ S(G, Aut(G)). Hence, our stance in this chapter is that G ≠ L(G) and g ∈ S(G, Aut(G)). It is worth noting that G ≠ L(G) ⇔ G possesses a non-trivial automorphism ⇔ |G| > 2. The motivation for studying the g-auto commuting probability of a finite group comes from the work in [11], where Pournaki and Sobhani studied an analogous notion.

Prg(G, Aut(G)) and various limits for it are obtained in the sections that follow. Pr (G, Aut(G)) also allows us to describe G. In the final part, we prove that, under autoisoclinism of groups, Prg (G, Aut(G)) is an invariant. Our article serves as the basis for this chapter.

A computing formula

The ratio is used to determine the g-auto commuting probability of G.

$$Pr_g(G, Aut(G)) = \frac{|\{(x,\alpha) \in G \times Aut(G): [x,\alpha] = g\}|}{|G||\,Aut(G)|} \qquad \text{(a)}$$

Note that if g = 1 then

$$Pr(G, Aut(G)) = \frac{|\{(x,\alpha) \in G \times Aut(G): [x,\alpha] = 1\}|}{|G||\,Aut(G)|} \qquad \text{(b)}$$

The set {α ∈Aut(G) : [x, α] = g} is denoted as Tx,g(G) for any values of x and g in G. Tx,1 = CAut(G)(x), as you indicated. In order to get the computation formula for Prg (G, Aut(G)), the following two lemmas are critical.

The Optimal G-Auto commuting Probability for a Complete set

The finite group G with two subgroups H and K of G such that H ⊆ K may be seen as an example. An element g from the set K is presented together with the probability that it is the auto commutator of a pair of elements chosen at random from H and Aut(K). The probability of G's subgroups H and K auto commuting is indicated by Pr_g (H, Aut(K)), and it is referred to as the generalized g-auto commuting probability of G. If H = The coverage is extensive, therefore Prg (H, Aut(K)) = Prg (G, Aut(G)). In addition, Pr_g(H,Inn(K)) = Pr_g(H, K) if we substitute Inn(K) for Aut(K), the inner automorphism group of K. We investigate numerous features and develop multiple computational equations for Pr_g (H, Aut(K)) in the following sections. Additionally, we find certain constraints for Pr_g (H, Aut(K)) and use it to describe various finite groups. It is our papers upon which this chapter is based.

We write S(H, Aut(K)) to donate {[x,α]: x ϵ H and α ϵ Aut(K)} and [H, Aut(K)]:= hS (H, Aut(K)) i We also write L (H, Aut(K)): = {x ϵ H: [x, α] = 1 for all α ϵ Aut(K)}. Note that L(G) = L (G, Aut(G)) and K(G)= [G, Aut(G)]. It is mentioning here that L (H, Aut(K)) is a normal subgroup of H contained in H ∩ Z(K) and L (H, Aut(K)) = $\cap_{\alpha \epsilon Aut(K)}$ CH(α), where CH(α): = {x ϵ H: [x, α] = 1} is a subgroup of H. Let CAut(K)(x): = {α ϵ Aut(K); α(x) = x} for x ϵ H and C_{Aut}(K)(H): = {α ϵ Aut(K): α(x) = x for all x ϵ H}. Then CAut(K)(x) is a sub group of Aut(K) and C_{Aut}(K)(H) = $\cap_{x \epsilon H}C_{Aut}$(K)(x). Note that Pr_g (H,Aut(K)) = 1 if and only if [H, Aut(K)] = {1} and g= 1 if and only if H= L (H, Aut(K)) and g = 1. Also, Pr_g(H, Aut(K)) = 0 if and only if g ϵ S (H, Aut(K)). Therefore, we consider H equals not to L (H, Aut(K)) and g ϵ S (H, Aut(K)) throughout this chapter.

Computing formulae

Probability that subgroups H and K of G satisfy the generalized g-auto commuting condition H ⊆ K is given by the ratio:

$$\mathrm{Pr}_g(H, \mathrm{Aut}(K)) = \frac{|\{(x, \alpha) \in H \times \mathrm{Aut}(K) : [x, \alpha] = g\}|}{|H||\,\mathrm{Aut}(K)|}$$

If g = 1 then

$$\mathrm{Pr}_g(H, \mathrm{Aut}(K)) = \mathrm{Pr}(H, \mathrm{Aut}(K)) = \frac{|\{(x, \alpha) \in H \times \mathrm{Aut}(K) : [x, \alpha] = 1\}|}{|H||\,\mathrm{Aut}(K)|}$$

Conclusion

The computational analysis of G-auto commuting probability in finite groups provides valuable insights into the structural properties and dynamics of these groups. By studying the likelihood that elements of a group commute under some automorphism, we gain a deeper understanding of the underlying symmetry and regularity of groups. This study contributes to the broader field of group theory, offering new methods and perspectives on the interplay between group elements and their automorphisms.

One of the key findings from this study is the potential for using G-auto commuting probability as an invariant measure for classifying groups. This approach allows us to identify groups with similar probabilities and explore their shared structural characteristics, which may aid in the classification and study of groups. Additionally, the computational methods and algorithms used in this analysis provide a practical framework for further research into more complex groups and their automorphism structures. Beyond group theory, the results have broader implications for areas such as coding theory and representation theory, where group symmetry is crucial. Understanding the interactions between group elements and automorphisms can inform these fields and lead to new applications and approaches. Moreover, this study establishes a foundation for future explorations of G-auto commuting probability across various groups, paving the way for new discoveries and advancements in the study of finite groups.

In conclusion, the study sheds light on the intricacies of G-auto commuting probability in finite groups and its significance in understanding group dynamics. It not only deepens our comprehension of group structure and behaviour but also opens up avenues for further research and applications in mathematics and related disciplines. By providing a computational approach to studying this probability, we offer researchers a valuable tool for exploring the complexities of group interactions under automorphisms.

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