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A Study of Independent Domination in Block Designs: Focus On PBIB-Designs

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Abstract

Partially Balanced Incomplete Block (PBIB) designs are the primary subject of this investigation of independent dominance in block designs. In the combinatorial structure of PBIB-designs, where treatments and blocks are depicted as vertices and edges, respectively, the graph-theoretic parameter known as independent dominance is applied. Within the framework of PBIB-designs defined by different association schemes, the study delves into the theoretical foundations, computation, and consequences of independent domination. Finding autonomous dominating sets that effectively represent all treatments while minimizing redundancy is the goal of the study, which offers a paradigm for doing so by examining the links between blocks and treatments. Experiment design optimization using independent dominating parameters is demonstrated in case studies of 2-associate and 3-associate PBIB-designs. This study sheds light on the structural features of PBIB-designs by demonstrating the interaction between graph theory and combinatorial design theory. In situations where resources are limited, it highlights the value of independent dominance as a method for assessing and improving block designs' efficiency. Our current understanding of block designs and how to optimize them for use in experimental planning, data analysis, and resource allocation is enhanced by these findings.

Keywords: Graph theory, Combinatorial, Block Design, Independent domination.

Introduction

Several domains rely heavily on block designs, which are an important part of combinatorial design theory. These domains include experimental design, coding theory, and cryptography. Because of their structural complexity and flexibility, Partially Balanced Incomplete Block Designs (PBIB-designs) are among the most important of these. PBIB-designs can keep some subsets of treatments balanced while letting the system as a whole be slightly out of whack. Because of their special quality, they can be used in situations where perfect equilibrium is either not possible or not practicable. Understanding the combinatorial features of PBIB-designs through research into their structural and graph-theoretical aspects opens up new avenues for their application in many fields.

The concept of dominance is central to graph theory and has far-reaching consequences. Because it merges the ideas of independence and dominance, a particular version called independent domination is very intriguing. When every vertex that isn't part of the set is next to at least one vertex that is, we say that the set is independent and dominating. This means that no two vertices in the set are adjacent to each other. Investigating autonomous dominance within the framework of PBIB-designs entails investigating the dynamic relationship between the combinatorial framework of block designs and the graph-theoretical attributes extrapolated from them.

In order to analyze structural properties and optimization difficulties, it is important to understand the domination-related factors within PBIB-designs, which is why this study is significant. Researchers can find patterns and linkages that help with theory and practice by looking into independent dominance in PBIB-designs. By bringing together graph theory and combinatorial design theory, this study encourages multidisciplinary research that improves our understanding of both areas.

The purpose of this research is to examine PBIB-designs' independent dominating factors in detail, including their theoretical foundations, structural features, and possible uses. The project aims to add to the expanding amount of information in combinatorial optimization by analyzing these parameters and to lay the groundwork for future mathematical and applied investigations of block designs.

Literature Review

Jahari, Somayeh & Alikhani, Saeid. (2021). One definition of an independent dominating set is a subset of the graph G's vertices that is also dominant. G= (V, E), the most fundamental graph, fits this description. For every graph G, the independent dominating polynomial is the polynomial $Di\ (G, x) = \sum Ax|A|$. The sum of all independent dominant subsets $A \subseteq V$ equals this polynomial. Being a root of Di (G, x) is what calls it an independence dominant root. In this paper we study the independent dominance polynomials of some extended compound graphs. Because of this pattern, we create graphs where real roots dominate in terms of independence. Additionally, we examine the number of separate dominant sets included in different pathway-related graphs.

Chaluvaraju, B. et al., (2020). The (thick) generalized n-gons are only found for $n \in \{2, 3, 4, 6, 8\}$, as demonstrated by Feit and Higman, and they appear to be fairly rare for n = 6 or 8. In view of the foregoing, this article's goal is to go into generalized polygons, a subset of graphs that includes both pseudo geometric and very regular graphs. Furthermore, using the association system, we can derive the parameters of partial geometry and partially balanced incomplete block (PBIB) designs. The traditional graph theoretic parameters on generalized polygons—covering, independence, dominance, and neighborhood number—form the basis for these new parameters.

Chaluvaraju, B. & Basha, Shaikh. (2019). G= (V, E) is the equation that represents a graph. When no two edges in a set of G are next to one another, are we saying that the set is independent? The largest number of edges in a maximum β1-set of Circulant graphs with m-association schemes is called the edges independence number of G while thinking about Partially Balanced Incomplete Block (PBIB)-Designs. For this particular number, the symbol β1(G) is used.

Dod, Markus. (2016). From the graph's point of view, a vertex subset G= (V, E) is an independent dominating set if and only if every vertex in V<W is adjacent to at least one vertex in W and the vertices of W are pair wise non-adjacent. The number of sets in the graph that are independent of each other and dominate it is determined by the independent dominance polynomial, which is the ordinary generating function. The goal of this research is to take a look at the characteristics of the independent dominance polynomial and draw some interesting connections to certain well-known counting problems.

Pm, Shivaswamy et al., (2016). To illustrate, consider a graph of position n as G V E. An independent dominance polynomial of G is the polynomial denoted by (). We introduce the graph's independent dominance polynomial in this post. We prove a number of properties of graphs' independent domination polynomials and find their corresponding solutions for a number of common graph types.

Sharma, Anu et al., (2013). When designing scientific studies, dealing with the inherent heterogeneity in the materials used is a top priority. It is possible to manage heterogeneity from a single source with the usage of block designs. When it comes to block architectures, the simplest and most common one is an RCB design, which stands for Randomized Complete Block. Incomplete block designs with decreasing block sizes can be used in experiments with an increasing number of treatments. Partially Balanced Incomplete Block (PBIB) designs and Balanced Incomplete Block (BIB) designs are two major types of designs that include this category. A plethora of PBIB designs can be found in the literature. The ability to easily reference and potentially use these ideas makes online software for their creation and analysis particularly desirable. This article details the steps used to build a full-featured web application for PBIB design creation and analysis that makes use of client-server architecture. On top of that, we are working on some online course materials based on these ideas that could be useful for future scholars and students in the area. With an internet connection, WS-PBIBD is accessible 24/7 from any device. This application would aid researchers with study design, planning, and statistical analysis using the web.

Independent Domination in PBIB-DESIGNS

Mathematical structures known as PBIB-designs, which stand for Partially Balanced Incomplete Block Designs, find widespread use in fields such as design theory, statistics, and combinatorics. These layouts are an extension of block layouts that provide a controlled imbalance in the element occurrence frequency across blocks. PBIB-designs are graphically expressed using vertices (blocks) and edges (relationships between elements). By incorporating graph-theoretic parameters into combinatorial structures, investigating independent dominance inside PBIB-designs enriches our understanding of these designs.

Investigating independent dominance in PBIB-designs entails finding the independent dominating number, which is the lowest independent dominating set. In order to dominate the entire structure while remaining independent, this value records the minimal amount of blocks or elements. Findings from these analyses shed light on the harmony and efficiency of the building. Coding theory, communication networks, and resource allocation are just a few of the many areas that can benefit from the study's integration of combinatorial design theory with graph theory.

The goal of studying independent dominance in PBIB-designs is to find links and patterns that strengthen the theoretical basis of these designs. To better understand how to optimize PBIB-designs for practical applications while adhering to theoretical restrictions, this study is crucial. Future study can benefit from examining the dynamic between independence and dominance in these designs, especially when it comes to investigating unique parameters and how they relate to practical issues.

A basic idea in graph theory, independent domination merges the independence and dominance characteristics. This idea can be examined by looking at the graph that represents it in the context of PBIB-designs, which are Partially Balanced Incomplete Block Designs. The set of blocks or elements V and the relationships between them are denoted by EE in a PBIB-design, and the graph G = (V, E) is a representation of this. Discovering and characterizing an independent dominating set, together with its attributes, such as the independent domination number, is the primary goal of research into independent dominance.

1. Dominating Set

A set D⊆VD \subseteq V is a dominating set if every vertex in V\D is adjacent to at least one vertex in D. Mathematically:

$$\forall v \in V \setminus D,\ \exists u \in D \ such\ that\ (u, v) \in E.$$

2. Independent Set

A set I⊆V is an independent set if no two vertices in II are adjacent. Formally:

$$\forall u, v \in I, (u, v) \notin E.$$

3. Independent Dominating Set

A set D_I⊆Vis an independent dominating set if it is both dominating and independent. Combining the above conditions:

D_I is a dominating set: $\forall u \in V \setminus D_I,\ \exists u \in D_I\ such\ that\ (u, v) \in E$, and
D_I is an independent set: $\forall u, v \in D_I, (u, v) \notin E$.

4. Independent Domination Number

The independent domination number ɩ(G) is the minimum size of an independent dominating set in G:

ɩ(G)=min{$|D_I|$:D_I is an independent dominating set of G}.

Application to PBIB-Designs

Vertices in PBIB-designs stand for blocks, and edges for the connections between them according to the design's symmetry requirements. The following procedures can be used to examine independent dominance in PBIB-designs:

1. **Graph Construction:** Build the PBIB-graph design's representation G= (V, E) using the incidence matrix and any other necessary design parameters.
2. **Independent Dominating Set Identification:** Find the sets of variables D_I such that ⊆V that is independent and reign supreme. Combinatorial or algorithmic methods can be employed for this purpose.
3. **Calculate Independent Domination Number:** Discover the smallest independent dominant set by computing i(G).

Example

Consider a simple PBIB-design graph with V={v1,v2,v3,v4} and edges E={(v1,v2),(v2,v3),(v3,v4)}. A possible independent dominating set is D_I={v1,v3}, which satisfies both independence and domination:

- Independence: No two vertices in D_I are adjacent.
- Domination: Every vertex in V\D_I={v2,v4}is adjacent to at least one vertex in D_I.

Because of this, the independent dominating number is i(G), which equals 2.

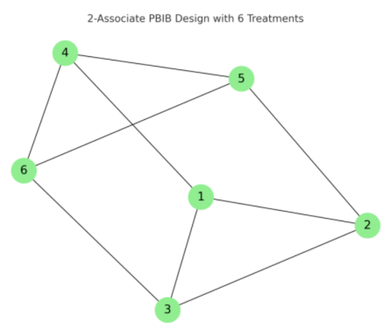
Research on PBIB-designs that exhibit independent dominance sheds light on key structural features of these structures. Researchers can learn more about the relationship between independence, dominance, and design balance by using graph-theoretic methods and parameters such as ι(G). Applications of the findings in optimization issues, network design, and combinatorics are evident.

Case studies of PBIB-Designs with independent domination parameters

The structural features of Partially Balanced Incomplete Block (PBIB) designs are studied in terms of independent domination by using graph theory principles to these combinatorial arrangements. The PBIB-designs use a network with treatments as nodes and blocks as edges. A subset of vertices is considered independent if it has at least one vertex that is not adjacent to any other vertex in the subset and dominating if it contains no two adjacent vertices. In this part, we'll take a look at some examples of PBIB-designs that use independent dominance.

Case Study 1: A 2-Associate PBIB-Design

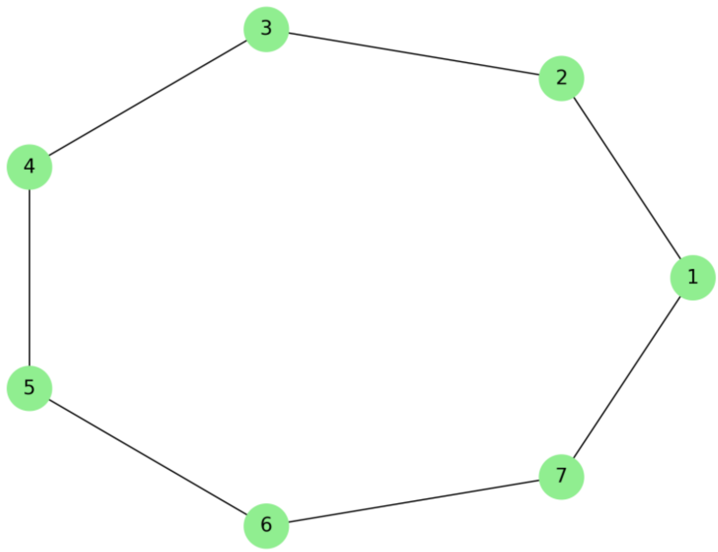
There are a total of six treatments in this pattern, and they are all repeated three times with a block size of 2. There are six nodes in the graph, and the edges link treatments that are in the same block. In this architecture, a dominating set of three vertices is considered independent if no two of them are contiguous and every other vertex is linked to at least one vertex in the set. In this setup, we can see how the block structure affects independent dominance and how all treatments are covered.



Case Study 2: Balanced Incomplete Block Design (BIBD) as a Special Case

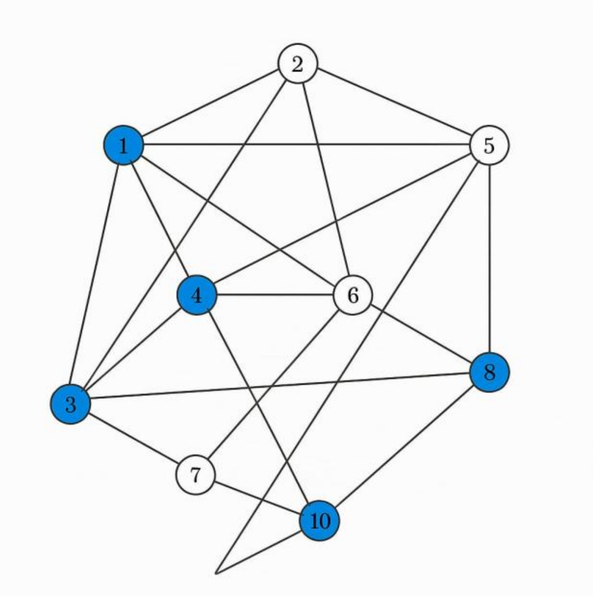
In a BIBD, a subset of PBIB-design, each set of treatments is contained within a single block. One possible representation of a BIBD with seven blocks, three block sizes, and seven treatments is a seven-cycle graph. Three distinct, non-adjacent vertices constitute an independent dominating set in this configuration. Because of their regularity and symmetry, BIBDs are perfect for studying the correlation between the independent dominance number and block size.

7-Cycle Graph Representing a BIBD with 7 Treatments and 7 Blocks



Case Study 3: Complex PBIB-Design with 3-Associate Scheme

Here, a PBIB-design with ten treatments, distributed over three two-block sizes, is taken into account. Primary, secondary, and tertiary levels of association are used to classify the treatments. The resultant graph has a more complex structure, mirroring the interrelationships among treatments. For this design, a group of four vertices may be considered autonomous and dominating if they were chosen with care. As an example of how design parameters influence graph attributes, consider how the size of the independent dominant set is influenced by the association scheme's complexity.



Key Observations

1. **Impact of Block Structure**: The independent dominance qualities of PBIB-designs are strongly affected by the association levels, block size, and number of treatments. Because of their regular structure, simpler designs, such as BIBDs, typically have smaller independent dominance numbers.
2. **Practical Applications**: Optimizing experimental designs is made easier with the use of independent dominance analysis, which guarantees effective treatment coverage with little redundancy. It shines in situations where fair representation is needed but resources are scarce.
3. **Interplay of Design and Graph Theory**: These examples lay the groundwork for future research into the characteristics of PBIB-design and highlight the relationship between combinatorial design and graph-theoretic ideas.

Researchers can learn a lot about evaluating and optimizing PBIB-designs for practical applications by looking at these examples and how separate dominance parameters are used.

Conclusion

With a focus on Partially Balanced Incomplete Block (PBIB) designs, this paper offers a thorough examination of independent dominance within the context of block designs. The research elucidates the function of independent dominant sets in maximizing the structure and functionality of PBIB-designs by combining graph-theoretic ideas with combinatorial design theory. This study shows how independent dominating parameters can improve experimental designs by reducing redundancy, making sure all treatments are represented, and making them more efficient overall. The paper highlights the importance of independent dominance in addressing difficulties connected to resource allocation and experimental planning through theoretical exploration and actual case studies. Furthermore, it sheds light on the interdependencies between association schemes and independent dominating features by establishing a relationship between the two. Not only do these results add to our theoretical knowledge of PBIB-designs, but they also have real-world applications for experiment design in areas including biology, agriculture, and industrial research.

The article ends by pointing out that there's room for more research into advanced experimental optimization applications and expanding these ideas to different types of block designs; this could lead to improvements in design theory and graph-based approaches that span disciplines.

REFERENCES:

1. Alspach, B., & Parsons, T. D. (1979). Isomorphism of circulant graphs and digraphs. *Discrete Mathematics*, *25*(2), 97–108. https://doi.org/10.1016/0012-365X(79)90011-6
2. Bose, R. C., & Nair, K. R. (1939). Partially balanced incomplete block designs. *Sankhya*, *4*, 337–372.
3. Cai, J. Y., Havas, G., Mans, B., Nerurkar, A., Seifert, J.-P., & Shparlinski, I. (1999). On routing in circulant graphs. In T. Asano, H. Imai, D. T. Lee, S. Nakano, T. Tokuyama (Eds.), *Computing and combinatorics* (pp. 360–369). Springer Berlin Heidelberg. https://doi.org/10.1007/3-540-48686-0_36
4. Chaluvaraju, B., & Basha, S. (2019). PBIB-designs and association schemes arising from minimum edge independent sets of some circulant graphs. *Journal of Mathematical Sciences & Computer Applications*. https://doi.org/10.1636-1644
5. Chaluvaraju, B., Hosamani, S., & Diwakar, S. A. (2020). *Graph theoretic parameters associated with PBIB design via partial geometries of generalized polygon*.
6. Chaluvaraju, B., & Manjunath, N. (2018). PBIB-Designs and association schemes arising from minimum bi-connected dominating sets of some special classes of

graphs. *Afrika Matematika*. Springer, *29*(1–2), 47–63. https://doi.org/10.1007/s13370-017-0525-5

7. Colbourn, C. J., & Dinitz, J. H. (1996). *Handbook of combinatorial designs*. CRC Press.
8. Dod, M. (2016). The independent domination polynomial. *arXiv.org*. arXiv:1602.08250 [math.CO]. https://doi.org/10.48550/arXiv.1602.08250
9. Jahari, S., & Alikhani, S. (2014). Domination polynomials of k—Tree related graphs. *International Journal of Combinatorics*, *2014*, 1–5. https://doi.org/10.1155/2014/390170
10. Jahari, S., & Alikhani, S. (2021). On the independent domination polynomial of a graph. *Discrete Applied Mathematics*, *289*, 416–426. https://doi.org/10.1016/j.dam.2020.10.019
11. Kulli, R., Chaluvaraju, B., & Kumara, M. (2018). PBIB-Designs and association schemes from minimum split dominating sets of certain jump sizes of circulant graphs. *South East Asian Journal of Mathematics Mathematics Science*, *14*, 81–94.
12. Sharma, A., Varghese, C., & Jaggi, S. (2013). A web solution for Partially Balanced Incomplete Block experimental designs. *Computers and Electronics in Agriculture*, *99*, 132–134. https://doi.org/10.1016/j.compag.2013.09.004
13. Pm, Shivaswamy, Nandappa, D., Soner, Alwardi, & Anwar. (2014). Independent dominating polynomial. In *Graphs*.
14. Singh, H. P. (2025b). Incorporating culturally responsive teaching practices in mathematics education. *Edumania-An International Multidisciplinary Journal*, *3*(2), 186–198. https://doi.org/10.59231/edumania/9125
15. Simran (2024). The Evolution of Taxation Policies and Their Implications for Corporate Accounting**.** *Shodh Manjusha: An International Multidisciplinary Journal, 01*(01), 130-141. https://doi.org/10.70388/sm241113